

## Borromean nuclei and three-body resonances

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**Abstract.** The interconnection between the resonances and virtual states of a three-body system and the resonances and virtual states in its two-body subsystems is investigated using the Borromean halo nucleus  $^{11}\text{Li}$  as a realistic example. The three-body core + neutron + neutron model is used, where the neutron-core interaction is fixed to reproduce a given  $s$ -wave virtual state and a  $p$ -wave resonance. The neutron-neutron interaction is then multiplied by a factor which is progressively changed from 0 to 1 and the resulting trajectories of the three-body states in the complex energy plane are investigated.

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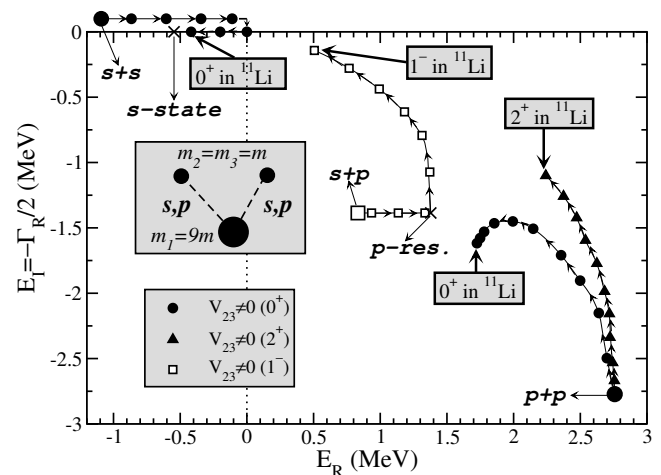
The properties of three-body systems are directly determined by the internal two-body structures. This statement is actually obvious as soon as only two-body interactions are involved in the three-body system. However, a direct and clean connection between the two-body and three-body properties has not been provided yet.

In this work we compute the three-body states by the complex scaled hyper-spherical adiabatic expansion method [1,2], which gives both bound states and resonances as (effectively) bound solutions of the Faddeev equations with energies independent of the scaling angle.

Starting from a trivial system in which the core has infinite mass and the two light particles do not interact with each other, we can first test the method, since the numerical calculations must reproduce accurately the expected trivial result. Secondly, this simple system is used as starting point in the calculations, being then possible to trace the different three-body states when more and more realistic features are introduced in the numerical calculations (*e.g.* finite core mass, interaction between the two light particles, finite particle and core spins, Pauli principle, and others).

In fig. 1 we show the three-body states for a system made by a zero-spin core with mass equal to  $9m$  (where  $m$  is the nucleon mass) and two neutrons with mass  $m$ .

The neutron-core  $s$ - and  $p$ -wave interactions are tuned to reproduce correspondingly a virtual (anti-bound) state at  $-0.54$  MeV and a resonance at  $1.4$  MeV with the width  $2.8$  MeV. Together with a realistic neutron-neutron interaction this parametrization of the neutron-core states gives an overall reasonable description of  $^{11}\text{Li}$  within the



**Fig. 1.** Evolution of the complex energies of different states in  $^{11}\text{Li}$  when the neutron-neutron interaction  $V_{23}$  is progressively introduced. The  $s$ - and  $p$ -wave neutron-core interaction is fixed to reproduce the virtual state and the resonance indicated in the figure. The real part of the energy,  $E_R$ , is plotted along the horizontal axes and the imaginary part,  $E_I$ , along the vertical axes. The core is assumed to have zero spin. The upper part of the lowest-energy  $0^+$  trajectory, shown outside of the figure, corresponds to a virtual (anti-bound) state, that is located on another (unphysical) sheet of the complex energy. Also the horizontal part of the  $1^-$  trajectory is located on the unphysical sheet of the complex energy.

$^9\text{Li}$  + neutron + neutron model and roughly corresponds to the spin-averaged structure of states in the neutron- $^9\text{Li}$  system [3,4]. The neutron-core states are marked with crosses in the figure.

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We use the neutron-neutron interaction from [5], which is a combination of central, spin-orbit, tensor, and spin-spin terms with a double-Gaussian shape fitted to the low-energy experimental data on  $s$ - and  $p$ -wave scattering. The neutron-neutron interaction is multiplied by a factor which is progressively changed from 0 to 1.

When the interaction between the neutrons is introduced, the different states (two  $0^+$ , one  $1^-$  and one  $2^+$ ) evolve as shown in the figure, such that the final points on each trajectory correspond to the states when the full neutron-neutron interaction is included. In particular, the lowest of the  $0^+$  states is a Borromean state, that is a bound three-body state where none of the two-body subsystems have bound states.

In the absence of the neutron-neutron interaction the lowest of the two  $0^+$  core + neutron + neutron states is a virtual (anti-bound) state, marked as  $s + s$  in the figure. It is located on the unphysical complex energy sheet at approximately twice the energy of the neutron-core virtual  $s$ -state, and contains predominantly neutron-core  $s$ -waves. The neutron-neutron interaction introduces an admixture of  $p$ -waves which reaches about 30% at the end point of the trajectory.

If the mass of the core were infinitely large, the three-body energy would have been precisely equal to the sum of the energies of the neutron-core virtual  $s$ -state. Since the mass of the core is finite the actual energy deviates from the sum. However, since the core is still much heavier than the neutrons this deviation is small and can hardly be seen in the figure.

As the neutron-neutron interaction is progressively introduced, this three-body state moves on the unphysical complex energy sheet towards zero. Having reached zero it enters the physical complex energy sheet and thus becomes a bound state. It then moves on the physical sheet towards its final destination at about  $-0.4$  MeV.

The  $1^-$  state shows similar behavior —it starts on the unphysical sheet at an energy, marked as “ $s + p$ ” in the figure, approximately equal to the sum of the energies of the neutron-core  $s$ -wave virtual state and  $p$ -wave resonance. At this point the state includes an equal admixture of the neutron-core  $s$ - and  $p$ -waves.

As the neutron-neutron interaction is introduced, it moves on the unphysical sheet towards the branching point at the energy of the  $p$ -wave neutron-core resonance, marked with a cross in the figure, where it enters the physical sheet and then moves on the physical sheet towards its final destination.

The trajectories for the highest  $0^+$  and the  $2^+$  states starts at the energy, marked as “ $p + p$ ” in the figure, approximately equal to twice the energy of the neutron-core  $p$ -wave resonance. At this point both these states are dominated by neutron-core  $p$ -waves.

The progressive introduction of the neutron-neutron interaction lifts the degeneracy of these states and makes them evolve along different trajectories on the physical complex energy sheet. It also introduces and admixture of  $s$ -waves into the  $0^+$  state.

For the sake of a cleaner picture we have assumed that only  $s$ -wave virtual states and  $p$ -wave resonances are present in the two-body subsystems. Thus the interactions in partial waves higher than  $s$  and  $p$  are assumed to be equal to zero. Since in Faddeev equations each partial wave of a Faddeev component is multiplied by a corresponding interaction [1], only  $s$ - and  $p$ -waves, where the interactions are nonzero, were included in each of the three Faddeev components.

Again, to have a clean picture we have in these calculations assumed that the spin of the core is equal zero as it is in some other halo systems like  ${}^6\text{He}$  and  ${}^{12}\text{Be}$ . Had the spin of the core been included, there would have been more neutron-core states and more trajectories would appear on the plot. Those trajectories would correspond to different spin-parities of the total core + neutron + neutron system and to different possible combinations of the neutron-core states. However, the general behavior of the new trajectories should be similar to one of the trajectories for our simplified system. We leave the classification of the effects of the finite core spin for a separate investigation.

Tracing the evolution of the three-body states permits one to visualize the features responsible for the existence of the different states, in particular the Borromean states. For a system like  ${}^{11}\text{Li}$  the Borromean ground state is due to the neutron-neutron interaction, while other systems like two heavy particles and a light one can have a Borromean state produced only by center of mass effects, even if the two heavy particles do not interact with each other. Furthermore, an understanding of the contribution from the different “realistic” features permits the use of the very schematic case as a starting point and make crude estimates of the spectrum of realistic three-body systems.

## References

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